

Can a highly virtual nucleon experience final state interactions in electron-nucleus scattering ?

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Abstract

We discuss how the virtuality of the struck particle may affect the final state interactions in electron-nucleus scattering. The extent to which short range correlations inhibit rescattering taking place within the range of the repulsive core of the NN interaction is quantitatively analyzed. The possible modifications of the nucleon-nucleon scattering amplitude associated with the virtuality is also studied, within the framework of a nonrelativistic model. The results suggest that the on shell approximation can be safely employed in the kinematical region relevant to the analysis of the available inclusive data at large momentum transfer and low energy loss.

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Inclusive electron-nucleus scattering in the quasielastic regime, at large momentum transfer q and low electron energy loss ω (corresponding to values of the Bjorken scaling variable $x = Q^2/2m\omega > 1.5$), has long been recognized as a unique tool to measure the high momentum components of the nuclear wave function or, equivalently, to get information on its behaviour at short interparticle distance in configuration space [1]. It is argued that, at large momentum transfer, electron-nucleus scattering reduces to the incoherent sum of elementary processes in which the electron scatters off individual nucleons, whose momentum \mathbf{k} and removal energy E are distributed according to the spectral function $P(\mathbf{k}, E)$, and the final state interactions (FSI) between the struck particle and the spectator nucleons are negligibly small. Within this picture, generally referred to as plane wave impulse approximation (PWIA), the inclusive cross sections are expected to exhibit a scaling behaviour in the variable y [2,3].

The available data [4–8], extending up to $Q^2 \sim 3 \text{ (GeV/c)}^2$ for heavy targets, show sizeable y -scaling violations, indicating that the PWIA regime has not been reached and FSI effects are still appreciable. The failure of PWIA to describe the data has been also consistently confirmed by the existing theoretical calculations of the inclusive cross sections, carried out using realistic spectral functions [9–13]. Therefore, a quantitative understanding of FSI appears to be needed to explain the measured cross sections and achieve a firm assessment of the longstanding issue of high momentum components and short range correlations in nuclei.

A theoretical treatment of inclusive electron-nucleus scattering at large momentum transfer including FSI effects, in which nucleon-nucleon (NN) correlations are consistently taken into account in both the initial and the final state within a microscopic many-body theory, has been proposed in ref. [10]. This approach accounts for the available data on different targets, ranging from deuteron to infinite nuclear matter¹ [10,12,13]. The main conclusion

¹the nuclear matter data have been obtained in ref. [7], fitting the cross sections for finite A to a

of refs. [10,12,13] is that most of the strength observed in the low energy loss wing of the cross section, say at $x > 1.4$, comes from processes in which the electron hits a nucleon of low momentum $k < k_F$, k_F being the Fermi momentum (in infinite nuclear matter at equilibrium density $k_F \sim .25 \text{ GeV}/c$), and the struck particle undergoes FSI with the $(A - 1)$ spectator nucleons.

In a recent paper [14], Frankfurt *et al* have criticized the approach of ref. [10] claiming that, since it does not take into account explicitly the virtuality of the struck particle, it is likely to overestimate the effect of FSI. If the struck nucleon is highly virtual, its lifetime, i.e. the time within which it has to interact to be brought back on shell, may become comparable with the range of the short range correlations ($\sim 1 \text{ fm}$), induced by the repulsive core of the NN force. On the other hand, NN correlations make the probability of finding a spectator to interact with at $t < 1 \text{ fm}$ very small. Hence, processes in which the struck particle is far off shell and undergoes FSI are expected to be suppressed. Moreover, the authors of ref. [14] argue that the amplitude describing the rescattering of the struck nucleon may also be affected by its virtuality, leading to a further quenching of FSI effects.

Whether a far off shell struck particle can undergo FSI obviously depends upon the underlying nuclear dynamics, which dictates the distribution in space of the spectators and the range of the rescattering amplitude. In this paper we follow the approach of ref. [10], in which the dynamical quantity relevant to the description of FSI is the eikonal propagator $U_q(t)$. The probability that the struck particle undergo FSI during the time t is given by $1 - U_q^2(t)$.

In this paper, the structure of $U_q(t)$ is quantitatively analyzed, with the aim of assessing the role of NN correlations in suppressing FSI of a far off shell, short lived, particle. We also estimate, within the framework of a nonrelativistic model, the difference between the imaginary parts of the nucleon self energy, obviously related to the imaginary parts of the

mass formula and singling out the volume term.

NN scattering amplitude, evaluated on and off shell in the energy and momentum range relevant to the analysis of the existing inclusive data.

In ref. [10] the inclusive electron nucleus cross section, i.e. the cross section of the process $e + A \rightarrow e' + X$, is written as a convolution integral:

$$\frac{d^2\sigma}{d\Omega d\omega} = \int d\omega' f_q(\omega - \omega') \left(\frac{d^2\sigma}{d\Omega d\omega'} \right)_{PWIA}, \quad (1)$$

where $(d^2\sigma/d\Omega d\omega)_{PWIA}$ denotes the PWIA cross section whereas the folding function $f_q(\omega)$, embedding all the information on FSI, is the Fourier transform of the eikonal propagator $U_q(t)$, defined as (for the sake of simplicity we will refer to the case of infinite nuclear matter at equilibrium density ρ):

$$U_q(t) = \exp \left\{ - \int_0^t d\tau \rho \int d^3r g(r) w_q(\mathbf{r} + \mathbf{v}\tau) \right\}. \quad (2)$$

In eq.(2), $g(r)$ is the nuclear matter radial distribution function, while \mathbf{v} denotes the velocity of the struck particle. The function $w_q(\mathbf{r})$ is related to the imaginary part of the NN scattering amplitude for incident momentum $\mathbf{p} = \mathbf{k} + \mathbf{q} \sim \mathbf{q}$, which is known to be dominant at large q :

$$w_q(\mathbf{r}) = \frac{2\pi}{m} \int \frac{d^3p'}{(2\pi)^3} e^{i\mathbf{p}'\mathbf{r}} \text{Im} f_q(\mathbf{p}') \quad (3)$$

The calculation of $U_q(t)$ according to eqs.(2)-(3) requires two basic ingredients: i) $g(r)$, yielding the probability that the struck particle hits one of spectators after travelling a distance r , and ii) the amplitude $f_q(\mathbf{p}')$, describing the scattering process between the struck nucleon and the spectators.

The results of ref. [10,12,13], obtained using distribution functions generated from realistic nuclear wave functions and the free space amplitudes extracted from NN scattering data [15], show that FSI is indeed significantly suppressed at small t on account of short range correlations. This feature is illustrated in fig.1, where the eikonal propagator of a nucleon of momentum $p = 1.95 \text{ GeV}/c$, evaluated with and without inclusion of correlation effects (i.e. setting $g(r) = 1$), are compared. It has to be kept in mind that the tails of the folding function $f_q(\omega)$ at $|\omega| > 0.3 \text{ GeV}$ are only sensitive to the shape of $U_q(t)$ at $t < 1 - 1.5 \text{ fm}$.

The calculations of refs. [10,12,13] also show that, in spite of the quenching produced by NN correlations, the inclusion of FSI produces a sharp rise of the cross section at $x > 1.4$, with respect to the PWIA predictions, whose driving mechanism can be readily understood. In the kinematical conditions under discussion, the PWIA nuclear response, defined as:

$$R(\mathbf{q}, \omega) = \int d^3k \, dE \, P(\mathbf{k}, E) \times \delta(\omega - E - \sqrt{|\mathbf{k} + \mathbf{q}|^2 + m^2} + m) \\ \simeq \frac{d^2\sigma(e + A \rightarrow e' + X)}{d^2\sigma(e + N \rightarrow e' + X)} \quad (4)$$

drops by three orders of magnitude as x goes from 1 to 2. As a consequence, even a few percent deviation from unity of $U_q(t)$ at short t ($t < 1 - 1.5 \, fm$), resulting in a tiny positive tail in $f_q(\omega)$ at large $|\omega|$ ($|\omega| \sim 0.3 - 0.5 \, GeV$), dramatically affects the folded cross section, moving strength from the quasi free bump ($x \sim 1$) to the low ω region ($x > 1.5$) according to eq.(1).

The above argument can be rephrased in a somewhat more physical language. The PWIA response in the region of the quasi free peak is dominated by processes in which the electron hits a slow nucleon, whereas the strength at $x > 1.5$ comes from scattering off nucleons of high momentum ($k \gg k_F$), whose probability is strongly suppressed. The results of ref. [10,12,13] essentially show that the dominance of low momentum nucleons in the nuclear ground state is so strong that, even if the probability of FSI at short t is quenched by NN correlations, scattering off slow nucleons undergoing FSI over a timescale of the order of $1 \, fm$ is much more likely to occur than scattering off fast nucleons. In fact, this turns out to be the leading mechanism responsible for the inclusive cross section at $x > 1.5$, as illustrated in fig.2. The PWIA nuclear matter response at incident energy $3.6 \, GeV$ and scattering angle 30° (dash-dot line), corresponding to $Q^2 = 2.3 \, (GeV/c)^2$ at the quasi free peak ($x = 1$), is shown as a function of x and compared to the results of the approach of ref. [10]. The solid line corresponds to the full calculation, whereas the dashed line has been obtained including in the calculation of FSI only the contributions coming from nucleons of initial momentum $k < k_F$, which appear to provide about 90% of the response at $x > 1.4$.

To make clear that $R(\mathbf{q}, \omega)$ at $x > 1.5$ is *only* sensitive to FSI occurring within $\sim 1 \, fm$,

in fig. 2 we also show the response evaluated from eq.(1) with a $U_q(t)$ which completely inhibits FSI at $t < r_c$, with $r_c \sim 1.1 \text{ fm}$. This $U_q(t)$, shown by the diamonds in fig.1, has been obtained using a steplike distribution function, $g(r) = \theta(r - r_c)$, with r_c fixed by the normalization, and a zero range NN amplitude in eqs.(2) and (3), respectively. It appears that, at $x > 1.5$, the diamonds come very close to the PWIA curve, showing that by artificially inhibiting the rescatterings occurring within $\sim 1 \text{ fm}$ one kills the whole FSI effect.

Realizing that the inclusion of *realistic* NN correlations does not necessarily results in a vanishing probability of FSI at $t < 1 \text{ fm}$ is very important since, according to the estimates of ref. [14], this is the relevant timescale for FSI to occur in processes involving nucleons of low initial momentum at Q^2 of a few GeV and low energy loss. Assuming that NN correlations totally inhibit FSI at $t < 1 \text{ fm}$, would therefore rule out scattering off low momentum nucleons followed by FSI as the relevant mechanism to move strength from the quasi free peak to the large x region. The results of refs. [10,12,13] seem to indicate that this is not the case.

The question still remains, however, whether our calculation of the eikonal propagator is strongly biased by the use of the on shell approximation for the rescattering amplitude, i.e. whether the small t behaviour of $U_q(t)$ is strongly affected by the differences between the free space scattering amplitude, employed in the calculations, and the amplitude describing the scattering of a far off shell nucleon. This problem has been first raised in ref. [16].

Very little is known about the off shell behaviour of the NN scattering amplitude, or more generally of the nucleon self-energy, in the relativistic regime relevant to the understanding of the inclusive data at high momentum transfer. Therefore, one has to rely on the guidance provided by nonrelativistic models.

The authors of ref. [16] argue that the imaginary part of the self energy of an off shell nucleon of momentum \mathbf{p} , related to the NN scattering t -matrix via

$$\text{Im } \Sigma(p, E) \sim \int \frac{d^3 p'}{(2\pi)^3} n(\mathbf{p}') \langle \mathbf{p}, \mathbf{p}' | \text{Im } t(E) | \mathbf{p}, \mathbf{p}' \rangle, \quad (5)$$

where $n(\mathbf{p}')$ denotes the momentum distribution, can be sizeably reduced with respect to its on shell value $Im \Sigma(p, p^2/2m)$. As an example, they quote the result of ref. [17], where the dependence upon p and E of the self energy of a dilute hard sphere Fermi gas has been analyzed. The calculations of ref. [17] show that for $E \sim E_F$, $E_F = k_F^2/2m$ being the Fermi energy, and $p > 3k_F$, $Im \Sigma(p, E)$ vanishes due to energy and momentum conservation. It has to be pointed out, however, that the argument of ref. [17] does not apply to the kinematical domain relevant to the study of FSI in electron nucleus scattering, corresponding to nucleon momenta $p \sim q \sim 2 \text{ GeV}/c$ and energy $E \sim \omega_q/2$, with $\omega_q \sim q^2/2m$.

In order to get some insight in the behaviour of $Im \Sigma(p, E)$ in the relevant kinematical region, we have carried out a calculation following the approach of ref. [18]. $Im \Sigma(p, E)$ has been evaluated for infinite nuclear matter including the second order Feynman diagram generally referred to as polarization graph, which has been shown to be responsible for most of the energy dependence [17], and using a Yukawa interaction. It has to be stressed that this somewhat oversimplified treatment is expected to be adequate for the purpose of the present study, since the energy dependence of $Im \Sigma(p, E)$ is dominated by the phase space available for the decay of a quasiparticle of momentum p into a two-particle one-hole state.

In fig.3 we show the results of our calculation, in the form of the ratio $Im \Sigma(p, p^2/4m)/Im \Sigma(p, p^2/2m)$, as a function of the nucleon momentum p . It appears that the deviation from unity rapidly decreases with p for p larger than $1 \text{ GeV}/c$, and becomes less than 5% at $p \sim 2 \text{ GeV}/c$, where the approach of ref. [10] is expected to be applicable.

A similar conclusion has been reached by Rinat and Tarragin in ref. [19], where the off shell t -matrix for a nonrelativistic potential model has been found to approach the on shell one for momenta around $2 \text{ GeV}/c$.

The different results obtained in ref. [16], whose authors find a large off shellness effect for momenta $p \sim 1 - 2 \text{ GeV}$, have probably to be ascribed to the particular parametrization of $Im \Sigma(p, E)$ employed in their calculations:

$$Im \Sigma(p, E) = Im \Sigma(\sqrt{2mE}, E) e^{-\frac{b^2}{4}(p^2-2mE)}. \quad (6)$$

Eq.(6) has been originally proposed in ref. [20] on the basis of an analysis of low energy nucleon-nucleus data. However, the authors of ref. [20] explicitly state that their prescription is not meaningful for momenta much larger than b^{-1} . Hence, extending its use to proton momenta in the GeV/c region, using at the same time b values corresponding to b^{-1} in the range $0.2-0.4 GeV/c$, needed to get a reasonable fit of the data, seems to be hardly justified.

In conclusion, the results of the present paper indicate that the treatment of FSI proposed in ref. [10], in which correlation effects are taken into account within a highly realistic many-body approach and the rescattering process is described in the on shell approximation, is consistent and applicable in the kinematical domain covered by the available inclusive data at large q and low ω . The fact that in this regime a struck nucleon of low initial momentum is sizeably off shell does not appear to affect the main conclusion of refs. [10,12,13], where the strength at $x > 1.4$ was ascribed mostly to electron scattering off slow nucleons undergoing FSI.

Nonrelativistic many body calculations suggest that the possibility that the eikonal propagator be strongly affected by the on shell approximation for the rescattering amplitude is very unlikely at $q > 1.5-2 GeV/c$. Furthermore, it appears that at $Q^2 \sim 2-3 (GeV/c)^2$ and $x \sim 2$, FSI occurring within the range of NN correlations, far from being totally inhibited, contribute about 90% of the response, while rescattering processes taking place over a longer time scale do not play a significant role.

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FIGURES

FIG. 1. Eikonal propagator of a nucleon of momentum $p = 1.95 \text{ GeV}/c$ in infinite nuclear matter. The solid line shows the results of the full calculation, whereas the dashed line has been obtained disregarding the effect of short range NN correlations. The diamonds show the results of the model calculation described in the text.

FIG. 2. Nuclear matter response function at incident energy $E = 3.595 \text{ GeV}$ and scattering angle $\theta = 30^\circ$, corresponding to $Q^2 = 2.3 (\text{GeV}/c)^2$ at the quasifree peak. Dot-dash line: PWIA; solid line: full calculation including the FSI; dashed line: FSI included only for nucleons of initial momentum $k < k_F$; diamonds: same as the dashed line, but with the FSI calculated using the $U_q(t)$ represented by the diamonds in fig.1.

FIG. 3. Ratio $\text{Im } \Sigma(p, p^2/4m)/\text{Im } \Sigma(p, p^2/2m)$, in infinite nuclear matter at equilibrium density, shown as a function of the nucleon momentum p .

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